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Transition to turbulence in pipe flow for water and dilute solutions of polyethylene oxide

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An experimental study of the transition from laminar to turbulent flow in a long 0·248 in. I.D. pipe is reported for both water and dilute water solutions of polyethylene oxide which exhibit turbulent flow drag reduction (the Toms phenomenon). The drag-reducing solutions, ranging in effectiveness from near zero to the maximum attainable, are observed to undergo transition in a similar way to the Newtonian solvent in that the solutions exhibit intermittency and the growth rates of the turbulent patches are essentially equal to those of the pure solvent. The growth rate of turbulent patches indicates that drag reduction is associated with the small-scale structure of the turbulence near the pipe wall while patch growth is associated with the larger-scale turbulence in the outer flow. For low-disturbance pipe inlet conditions the strong drag-reducing solutions are observed to undergo transition at lower Reynolds numbers than the pure solvent.

1. Introduction

Since the phenomenon of intermittency (alternating patches of laminar and turbulent flow with time) in the process of pipe flow transition was reported by Reynolds (1883), the studies of Prandtl & Tietjens (1934), Rotta (1956), Lindgren (1957) and Coles (1962) have identified the essential features of this phenomenon. Intermittency is now understood to be an integral part of the laminar to turbulent transition occurring as the last stage of transition prior to fully developed turbulence.

For high-disturbance pipe-inlet conditions, transition occurs over a Reynolds number range from about 2100 to 2800. At the lower end of this range patches of turbulent flow are observed at distances somewhat removed from the pipe inlet and have a length of the order of 20 pipe diameters and a zero growth (elongation) rate, the latter leading to a flow which is statistically stationary with distance from the pipe inlet. As the Reynolds number is increased the turbulent-patch birth rate increases and the patches acquire a small positive growth rate. This results in the merging of adjacent patches and the establishment of fully developed turbulent pipe flow. For low-disturbance inlet conditions patches of turbulence are first observed at higher Reynolds numbers and possess a large

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positive growth rate. At pipe locations far removed from the pipe inlet the flow is fully turbulent and independent of the frequency of birth of patches.

Since the discovery of the phenomenon of drag reduction with dilute polymer solutions by Toms (1948) there appear to have been no detailed investigations of the transition to turbulence with these solutions in pipe flow or other geometries. For the case of high-disturbance pipe-inlet conditions several investigations have been concerned with the effect of dilute polyethylene oxide solutions on the transition Reynolds number range. While Castro & Squire (1967) and White & McEligot (1970) have reported that some solutions cause an increase in the transition Reynolds number range relative to that of the pure solvent (delay in transition), Virk *et al.* (1967) found that similar solutions may never delay transition. The present study was undertaken to determine whether polymer solutions, which significantly reduce the friction factors for fully developed turbulent pipe flow and hence affect the turbulence in a fundamental manner, also alter the transition process.

2. Experimental procedure

Transition tests were conducted in a 0·248 in. I.D. by 1700-diameter pipe constructed of smooth seamless brass tubing joined in sections by flanges. Pipe eccentricity and tolerance on diameter was measured to be of the order of 0·001 in. with possible maximum flange misalignment slightly greater. The pipe was provided with three pressure-drop measuring stations at 912, 1302 and 1541 diameters from the pipe inlet to permit the variation in patch characteristics with pipe distance to be measured. The pressure tap separation at each station was held at 16 diameters to prevent more than one patch being between the taps at a given time. This would have obscured the pressure-drop versus time signal. Two variable-reluctance differential pressure transducers (0–1 psi) were connected to two of the pressure-drop measuring stations with the pressure-drop signals displayed on a two-beam oscilloscope or X–Y recorder. The tubing connecting the pressure taps to the transducers was completely filled with water. This, together with the small volume flow through the pressure taps required for full-scale transducer diaphragm deflexion (0·0003 in.^{3}) produced a fast time response. Although the rated high frequency limit of the transducer equipment was known to be 1000 hertz, the frequency response of the system was not known. Practically, the frequency response was high enough to provide clear identification of turbulent patch interfaces.

The tests were conducted at room temperature (21–24 °C) and the mean velocity determined by weighing fluid collected at the pipe outlet over a timed interval. The net accuracy of the velocity and pressure-drop measurement was 2 % since this was the agreement obtained when distilled-water results were compared with the laminar and Blasius smooth-pipe turbulent-friction factors. Flow-rate was controlled by pressurization of the inlet tank and/or throttling with a valve at the pipe exit. Both a low-disturbance pipe inlet (smooth nozzle) and high disturbance inlet (squared-off pipe) were employed. Measurements of the disturbance level introduced by the two pipe inlets were not taken nor is it possible to estimate this level. The high-disturbance inlet produced transition with water in the 2500–2800 Reynolds number range, whereas the corresponding values for the low disturbance inlet were 8000–11 000.

Dilute solutions of polyethylene oxide were prepared using various unfractionated molecular weight blends of Polyox manufactured by the Union Carbide Corporation. Polymer concentration was limited to the 'dilute solution' range in which the ratio of polymer molecule centre-to-centre separation to polymer coil diameter was order 1 or greater (Paterson 1969). Calculations of Reynolds numbers were based upon measured zero shear rate solution viscosities. Further details of the apparatus and pipe inlets in addition to viscosity, molecular weight and drag-reduction measurements are described elsewhere (Paterson & Abernathy 1970).

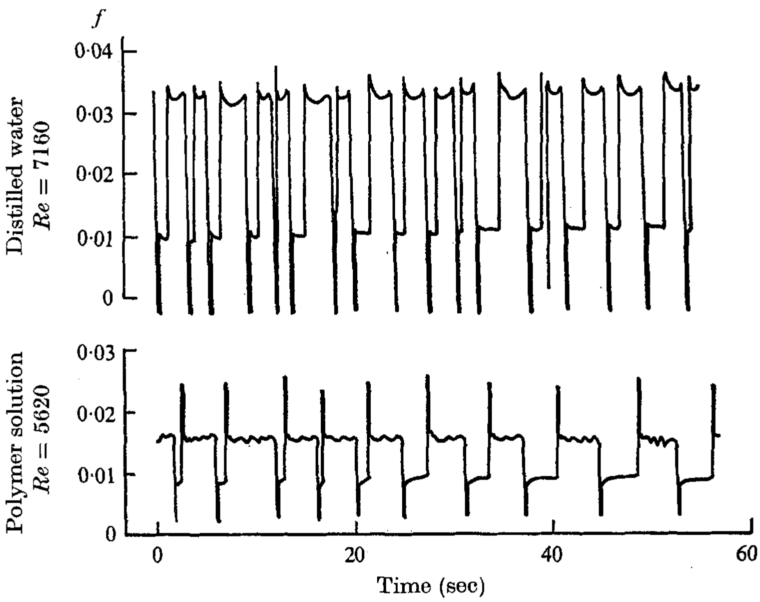

FIGURE 1. Pressure transducer output versus time at 912 diameters from the pipe inlet for both distilled water and a 50 p.p.m. solution of polyethylene oxide, WSR-205, weight average molecular weight measured to be approximately 1×10^6. Smooth-nozzle pipe inlet.

3. Distilled-water and polymer-solution intermittency

Figure 1 displays intermittent flow during transition under low inlet-disturbance conditions (smooth-nozzle inlet) for both distilled water and an effective drag-reducing solution, 50 p.p.m. WSR-205; figure 3 gives the friction factors obtained with this solution. The polymer solution exhibits intermittency, behaving in essentially the same way as the Newtonian solvent except that the Reynolds number range for transition is lower, an observation considered later. All polymer solutions tested, ranging in concentration from 0·03 to 75 p.p.m. (1 p.p.m. = 1 g of dry polymer per 10^6 c.c. of distilled water), weight average molecular weight from $2{\cdot}5 \times 10^5$ to 8×10^6 and consequent extent of drag reduction from near zero to the maximum obtainable (adherence to Virk's (1967) maximum drag-reduction asymptote) displayed turbulent patches during transition as indicated by the pressure drop signal. Further confirmation was offered by the jet issuing from the pipe outlet which was observed to alternate between being smooth (laminar) and wrinkled (turbulent)† and fluctuate up (laminar) and down (turbulent) during transition. It should be noted that turbulent patches are difficult to detect for strong drag-reducing solutions since the turbulent friction factors for these solutions differ little from the friction factors for laminar flow at the transition Reynolds number.

4. Turbulent patch velocities

The velocities of the front and trailing edges (laminar–turbulent interfaces) of turbulent patches were measured by timing the transit of the front or trailing edge over the distance between the first and third or second and third pressure-drop stations. The times were determined from oscilloscope photographs or recorder plots of the transducer outputs at the two stations. The location of the mean velocity determination was 1226 diameters when the first and third stations were used and 1421 diameters for the second and third stations. Patch velocities with both arrangements were found to be equal, suggesting negligible patch interface acceleration or deceleration. The mean velocity and patch velocity were measured simultaneously to permit comparison. Figure 2 is a plot of the ratio of the front or trailing edge velocity U to the mean velocity V as a function of Reynolds number. The Reynolds number for intermittency was varied by the use of different inlets and inlet tank quieting times before testing. Data are also included for single patches generated by a deliberate disturbance at the pipe inlet.‡ This permitted measurement of patch velocities at Reynolds numbers below that at which intermittency normally occurs with the smooth nozzle inlet.

Figure 2 extends Lindgren's patch velocity data (dashed lines) to higher Reynolds numbers and includes data taken with distilled water and polymer solutions of varying drag-reducing effectiveness (friction factor plots are shown in figure 3). The solid circle shown at a Reynolds number of 2150 represents equal patch velocities obtained for the front and trailing edges of the patch with distilled water. At this Reynolds number therefore a state of laminar–turbulent equilibrium exists in which the patch maintains its constant length of about 20 diameters as it traverses the pipe. The front and trailing-edge velocities of $0{\cdot}92V$ mean there is a net mass flow through the patch from rear to front with turbulence leaving the front of the patch on centre-line of the pipe (and decaying in a zero mean shear field) and laminar flow entering the rear (and becoming turbulent). Above a Reynolds number of about 2300 the patch trailing-edge velocity lags that of the front so that a positive patch growth rate exists. Once

† Observable only when the valve at the pipe exit which obscures this phenomenon is removed.

‡ A disturbance is generated in the smooth nozzle inlet when the flow is started impulsively, the intensity of which can be increased by increasing the inlet tank pressure while setting a throttle valve at the pipe outlet to yield the same steady state flow-rate. This disturbance, if of sufficient intensity, produces a single turbulent patch.

born, a patch will continue to grow until, sufficiently far downstream, neighbouring patches merge to form fully developed turbulence.

Figure 2 demonstrates that the polymer solutions, ranging in drag-reducing effectiveness from small (1 p.p.m.) to near the maximum obtainable (50 p.p.m.), have essentially the same patch-velocity ratios and hence patch growth rates as distilled water. This occurs even though the polymer solutions possess a significantly lower wall shear stress and hence friction velocity at transition.

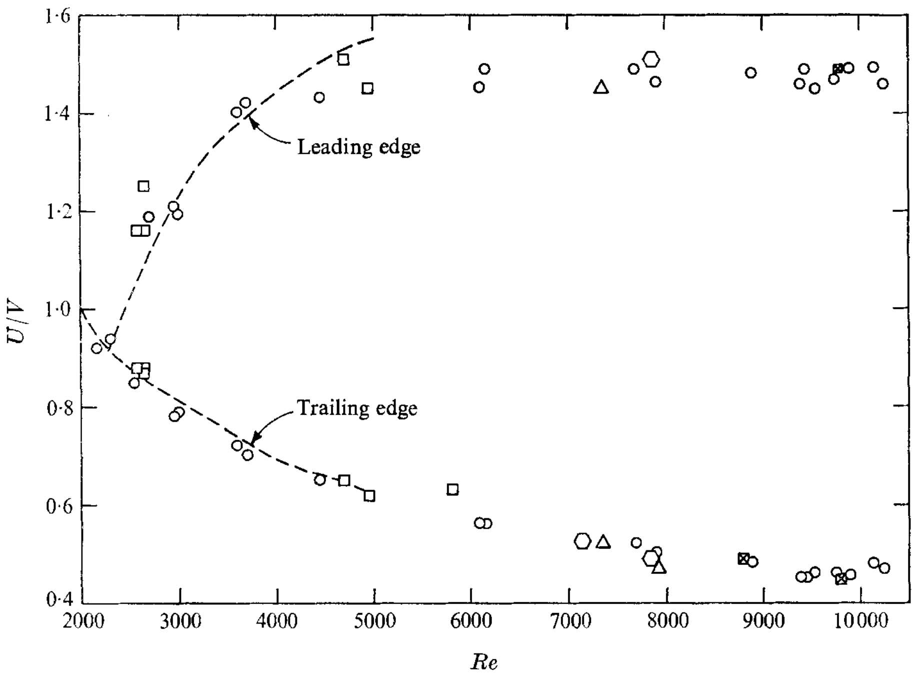

FIGURE 2. Turbulent-patch interface velocity to mean velocity ratio as a function of Reynolds number. ○, distilled water. Polyethylene oxide solution, WSR-205; □, 50 p.p.m.; ⬡, 40 p.p.m.; ⊠, 20 p.p.m.; △, 10 p.p.m., patch velocity data of Lindgren as given by Coles (1962). To obtain the data point at $Re = 2150$, inlet disturbance level was increased by inserting a $\frac{5}{8}$ in. long, 0·156 in. I.D. tube inside the 0·248 in. I.D. squared-off pipe inlet.

Since the small-scale turbulence structure near the wall is measured in units of the wall length scale ν/U_τ, where ν is the kinematic viscosity and U_τ the friction velocity, the data indicate that the patch growth rate does not depend on the small-scale wall structure but rather on the outer flow. The equality of growth rates for polymer solutions and water suggests that they have similar outer flow structures. This indirect reasoning then suggests that the polymer drag reduction is due to phenomena occurring near the wall. These conclusions are confirmed by Virk's 'onset hypothesis', in which the onset of drag reduction is found to depend on the ratio of polymer molecule coil size to wall length scale and by the measurements showing equality of the turbulence macro-scale in the outer flow region for the water solvent and polymer solution (Virk *et al.* 1967).

The agreement between Lindgren's patch velocity data and the above is considered good; both the original data of Lindgren (1957) and that presented here show considerable scatter in the front velocity measurements but low scatter for the trailing-edge velocity. The present experiment can be faulted in that the pipe system is operated at constant overall pipe pressure drop rather than constant flow-rate, thereby leading to flow-rate fluctuations. The relatively small scatter in the trailing-edge velocities, however, indicates that this is not the cause of the front scatter, the reason for which is unknown.

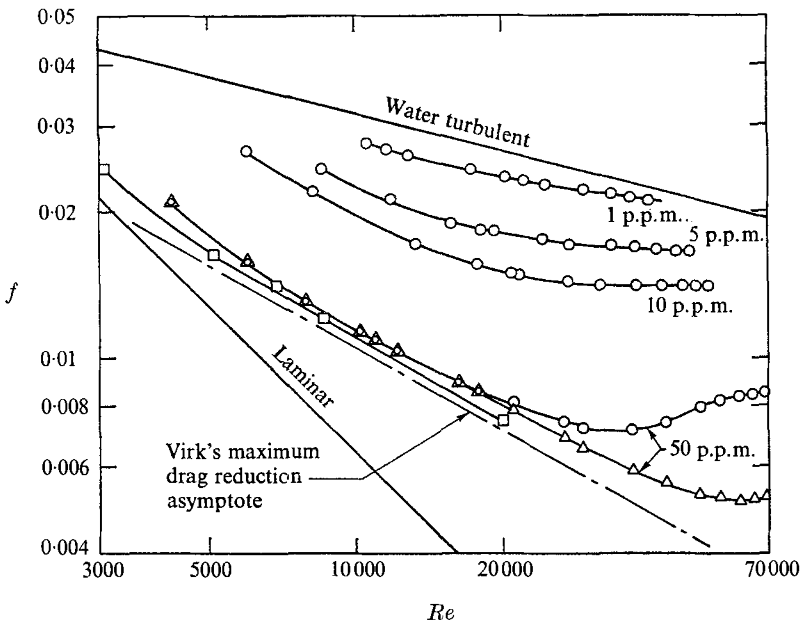

FIGURE 3. Pipe friction factor as a function of Reynolds number for various polymer solutions, obtained with a smooth nozzle inlet. ○, various concentrations of polymer WSR-205, molecular weight approximately 1×10^6; friction factors measured at 1541 diameters from the pipe inlet. △, 50 p.p.m. WSR-205, measured at 214 diameters. □, 50 p.p.m. WSR-301, molecular weight measured to be 8×10^6; friction factors measured at 1541 diameters. $dp/dx = f\rho V^2/2D$.

5. Effect of polymer solutions on transition Reynolds numbers

Figure 3 is a friction factor–Reynolds number plot for four concentrations of a moderate molecular-weight polymer (WSR-205, weight average molecular weight approximately 1×10^6) and a 50 p.p.m. solution of a high-molecular-weight polymer (WSR-301, weight average 8×10^6) obtained with the smooth nozzle low-disturbance pipe inlet. The left-hand end of each curve represents approximately the point at which transition to turbulence was observed as shown by intermittency in the pressure-drop signal.

By considering the WSR-205 data (circles) it is seen that as the concentration and hence extent of drag reduction increases, the turbulent transition tends to occur at lower Reynolds numbers. This trend was generally observed in the present study but with several test runs contradicting this. With a smooth nozzle inlet the transition Reynolds number is strongly dependent on inlet tank disturbances. Tank disturbances depend upon tank geometry, thermal convection currents, the manner of tank filling, the quieting time before testing, the level in the tank, the manner in which the flow is started, etc. Although these factors make it difficult to reproduce the inlet disturbance level, 50 p.p.m. solutions of WSR-205 consistently underwent transition between $Re = 4000$ and 6000 as the quieting time between tank filling and testing was varied from near zero to 15 h. The comparable range for distilled water was 8000–11000. Figure 4 is a plot of the average intermittency factor (fraction of time that the transducer output is turbulent) versus Reynolds number for both 50 p.p.m. WSR-205 and distilled water obtained at 1541 diameters from the pipe inlet. The polymer solution was quieted 15 h before testing to achieve a low tank-disturbance level.

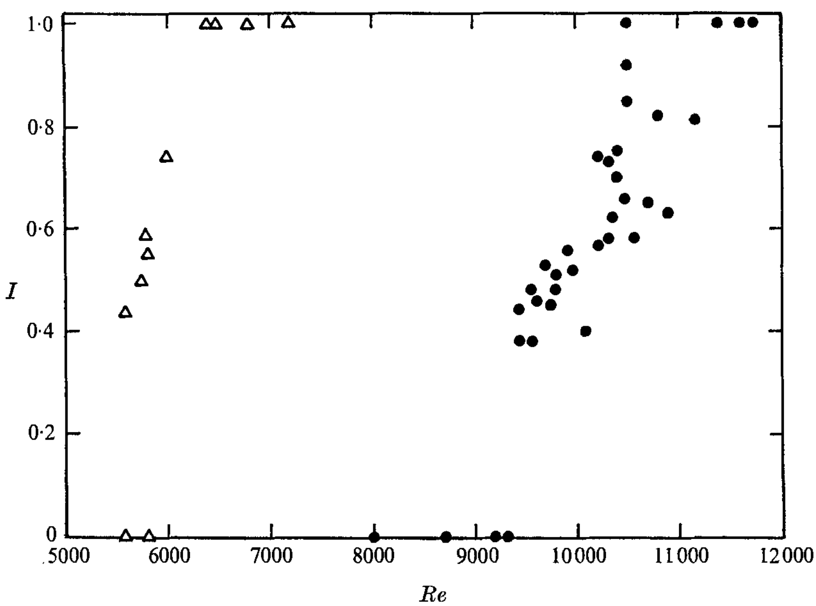

FIGURE 4. Average intermittency factor as a function of Reynolds number for polymer solution, 50 p.p.m. WSR-205 and distilled water, measured at 1541 diameters from smooth-nozzle pipe inlet. △, polymer solution quieted 15 h before testing; ●, quieted between 2 and 15 h. H_2O

The quieting time for water was varied from 2 to 15 h. However, the Reynolds number range for transition remained consistently higher than that for the polymer solution. The most clear-cut case studied was that of 50 p.p.m. WSR-301, a solution showing close agreement with Virk's maximum drag-reduction asymptote and thus representing the strongest possible drag-reducing solution. This solution invariably underwent transition between $Re = 2900$ and 3300 with the smooth-nozzle pipe inlet.

For high inlet-disturbance conditions (pipe equipped with squared-off pipe inlet), the transition Reynolds numbers for distilled water and the polymer solutions were within 400 of one another and therefore the effect of the polymer, if any, is small. This conclusion is in agreement with the findings of Virk *et al.* (1967) (the authors express some reservations in this conclusion) and in disagreement with the findings of Castro & Squire (1967) and White & McEligot (1970). There is no obvious reason for the difference in these results. However, some problems exist in comparing these studies. First, inlet-nozzle details and estimates of the reproducibility in the transition Reynolds-number range are not available for all tests. Second, some tests exceed the dilute-solution range (§ 2 above) so that the viscosity and hence Reynolds number cannot be calculated unequivocally; this arises from the shear dependence of solution viscosity, which has not been measured in most instances. Third, the friction factor–Reynolds number plot cannot be considered a reliable indicator of transition for polymer solutions. As shown in figure 3, the friction factors as measured at 1541 diameters diverge from those measured at 214 diameters above $Re = 17000$ for 50 p.p.m. WSR-205. This effect, which has been found to depend on polymer concentration and molecular weight, is due to degradation of the polymer solution (molecular scission) by the turbulent shear field (Paterson & Abernathy 1970). Although the polymer solution displayed turbulent patches and hence transition at $Re = 4200$, the friction factor plot at 1541 diameters could lead one to conclude incorrectly that the flow remained laminar up to $Re = 17000$ followed by a turbulent transition extending to a fully turbulent drag-reduced flow at $Re = 70000$.

The significant lowering of transition Reynolds numbers observed in the present study with strong drag-reducing solutions and a low-disturbance pipe inlet is surprising in that one might have expected the opposite effect. The results suggest that such solutions are less stable to small disturbances than the pure solvent. The issue is not clear however, for polymer solutions exhibit non-Newtonian behaviour (as shown by the high viscosity exhibited in irrotational flow fields (Lumley 1969) but low viscosity in rotational Poiseuille or Couette flow). Hence the laminar flow in the smooth nozzle inlet may vary amongst distilled water and the various polymer solutions, with other factors such as geometry and inlet tank-disturbance level held constant.

6. Conclusions

The experimental studies reported here indicate that the last stage of pipe flow transition, the intermittent flow régime, is similar for drag-reducing polyethylene oxide solutions and their Newtonian solvent. The rate of turbulent-patch growth appears to depend on the outer flow rather than the near wall region and it is in the latter region where polymer effects are appreciable. The need for further study of the effect of polymer solution variables on the transition Reynolds number range, preferably with a variable (and measured) level of inlet disturbance and constant flow-rate system, is also indicated.

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